# Package 'grpnet' 

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Type PackageTitle Group Elastic Net Regularized GLMs and GAMsVersion 0.5
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Description Efficient algorithms for fitting generalized linear and additive models with group elas-tic net penalties as described in Helwig (2024) [doi:10.1080/10618600.2024.2362232](doi:10.1080/10618600.2024.2362232). Imple-ments group LASSO, group MCP, and group SCAD with an optional group ridge penalty. Com-putes the regularization path for linear regression (gaussian), logistic regression (bino-mial), multinomial logistic regression (multinomial), log-linear count regression (pois-son and negative.binomial), and log-linear continuous regression (gamma and inverse gaus-sian). Supports default and formula methods for model specification, k-fold cross-validation for tuning the regularization parameters, and nonparametric regression via ten-sor product reproducing kernel (smoothing spline) basis function expansion.
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auto Auto MPG Data Set

## Description

Miles per gallon and other characteristics of vehicles from the 1970s-1980s. A version of this dataset was used as the 1983 American Statistical Association Exposition dataset.

## Usage

data("auto")

## Format

A data frame with 392 observations on the following 9 variables.
mpg miles per gallon (numeric vector)
cylinders number of cylinders: $3,4,5,6,8$ (ordered factor)
displacement engine displacement in cubic inches (numeric vector)
horsepower engine horsepower (integer vector)
weight vehicle weight in of lbs. (integer vector)
acceleration $0-60 \mathrm{mph}$ time in sec. (numeric vector)
model. year ranging from 1970 to 1982 (integer vector)
origin region of origin: American, European, Japanese (factor vector)

## Details

This is a modified version of the "Auto MPG Data Set" on the UCI Machine Learning Repository, which is a modified version of the "cars" dataset on StatLib.
Compared to the version of the dataset in UCI's MLR, this version of the dataset has removed (i) the 6 rows with missing horsepower scores, and (ii) the last column giving the name of each vehicle (car.name).
coef

## Source

The dataset was originally collected by Ernesto Ramos and David Donoho.
StatLib—Datasets Archive at Carnegie Mellon University http://lib.stat.cmu.edu/datasets/cars.data
Machine Learning Repository at University of California Irvine https://archive.ics.uci.edu/ml/datasets/Auto+MPG

## Examples

```
# load data
data(auto)
# display structure
str(auto)
# display header
head(auto)
# see 'cv.grpnet' for cross-validation examples
?cv.grpnet
# see 'grpnet' for fitting examples
?grpnet
```

coef Extract Coefficients for cv.grpnet and grpnet Fits

## Description

Obtain coefficients from a cross-validated group elastic net regularized GLM (cv.grpnet) or a group elastic net regularized GLM (grpnet) object.

```
Usage
    ## S3 method for class 'cv.grpnet'
    coef(object,
        s = c("lambda.min", "lambda.1se"),
        ...)
    ## S3 method for class 'grpnet'
    coef(object,
        s = NULL,
        ...)
```


## Arguments

object
Object of class "cv.grpnet" or "grpnet"

Lambda value(s) at which predictions should be obtained. For "cv.grpnet" objects, default uses the lambda that minimizes the cross-validation loss function. For "grpnet" objects, default uses s = object\$lambda. Interpolation is used for $s$ values that are not included in object\$lambda.
.. Additional arguments (ignored)

## Details

coef.cv.grpnet:
Returns the coefficients that are used by the predict.cv.grpnet function to form predictions from a fit cv.grpnet object.
coef.grpnet:
Returns the coefficients that are used by the predict. grpnet function to form predictions from a fit grpnet object.

## Value

For multinomial response variables, returns a list of length length(object\$ylev), where the $j$-th element is a matrix of dimension $c$ (ncoef, length(s)) giving the coefficients for object\$ylev[j].
For other response variables, returns a matrix of dimension $c$ (ncoef, length(s)), where the i-th column gives the coefficients for $s[i]$.

## Note

The syntax of these functions closely mimics that of the coef.cv.glmnet and coef.glmnet functions in the glmnet package (Friedman, Hastie, \& Tibshirani, 2010).

## Author(s)

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## References

Friedman, J., Hastie, T., \& Tibshirani, R. (2010). Regularization paths for generalized linear models via coordinate descent. Journal of Statistical Software, 33(1), 1-22. doi:10.18637/jss.v033.i01

Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. Journal of Computational and Graphical Statistics. doi:10.1080/ 10618600.2024.2362232

## See Also

print.coef.grpnet for printing coef.grpnet objects
predict.cv.grpnet for predicting from cv.grpnet objects
predict. grpnet for predicting from grpnet objects

## Examples

```
######***###### grpnet ######***######
# load data
data(auto)
# fit model (formula method, response = mpg)
mod <- grpnet(mpg ~ ., data = auto)
# extract coefs for regularization path (output = 12 x 100 matrix)
coef(mod)
# extract coefs at 3 particular points (output = 12 x 3 matrix)
coef(mod, s = c(1.5, 1, 0.5))
######***###### cv.grpnet ######****#####
# load data
data(auto)
# 5-fold cv (formula method, response = mpg)
set.seed(1)
mod <- cv.grpnet(mpg ~ ., data = auto, nfolds = 5, alpha = 1)
# extract coefs for "min" solution (output = 12 x 1 matrix)
coef(mod)
# extract coefs for "1se" solution (output = 12 x 1 matrix)
coef(mod, s = "lambda.1se")
# extract coefs at 3 particular points (output = 12 x 3 matrix)
coef(mod, s = c(1.5, 1, 0.5))
```

cv. compare

Compare Multiple cv.grpnet Solutions

## Description

Creates a plot (default) or returns a data frame (otherwise) that compares the cross-validation error for multiple cv.grpnet fits.

## Usage

```
cv.compare(x,
s = c("lambda.1se", "lambda.min"),
    plot = TRUE,
    at = 1:length(x),
    nse = 1,
```

```
point.col = "red",
line.col = "gray",
lwd = 2,
bwd = 0.02,
labels = NULL,
xlim = NULL,
ylim = NULL,
xlab = NULL,
ylab = NULL,
...)
```


## Arguments

x
S
plot
at
nse
point.col
line.col color for lines used to plot the standard error for the cv error.
lwd width of lines used to plot the standard error for the cv error.
bwd width of standard error bars in terms of proportion of range ( $x$ ).
labels labels for x -axis tick marks. Defaults to names $(x)$.
xlim axis limits for abscissa (x-axis)
ylim axis limits for ordinate (y-axis)
$\mathrm{xlab} \quad$ axis label for abscissa ( x -axis)
ylab axis label for ordinate (y-axis)
... additional arguments passed to plotting functions.

## Details

Default behavior creates a plot that displays the mean cv error $+/-1$ se for each of the requested solutions.
If the input $x$ is a single cv .grpnet object, then the function plots the lambda.min and lambda. 1 se solutions.
If the input x is a list of cv .grpnet objects, then the function plots either the lambda.min or the lambda.1se solution (controlled by s argument) for all of the input models.

## Value

When plot = TRUE, there is no return value (it produces a plot)
When plot = FALSE, a data.frame is returned with the mean cv error (and se) for each solution

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## References

Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. Journal of Computational and Graphical Statistics. doi:10.1080/ 10618600.2024.2362232

## See Also

plot.cv.grpnet for plotting cv error path (for all lambdas)
plot.grpnet for plotting regularization path (for single lambda)

## Examples

```
# load data
data(auto)
# LASSO penalty
set.seed(1)
mod1 <- cv.grpnet(mpg ~ ., data = auto, nfolds = 5, alpha = 1)
# MCP penalty
set.seed(1)
mod2 <- cv.grpnet(mpg ~ ., data = auto, nfolds = 5, alpha = 1, penaly = "MCP")
# SCAD penalty
set.seed(1)
mod3 <- cv.grpnet(mpg ~ ., data = auto, nfolds = 5, alpha = 1, penaly = "SCAD")
# compare lambda.min and lambda.1se for mod1
cv.compare(mod1)
# compare lambda.1se for mod1, mod2, mod3
cv.compare(x = list(mod1, mod2, mod3), labels = c("LASSO", "MCP", "SCAD"))
```

```
cv.grpnet Cross-Validation for grpnet
```


## Description

Implements k -fold cross-validation for grpnet to find the regularization parameters that minimize the prediction error (deviance, mean squared error, mean absolute error, or misclassification rate).

## Usage

```
    cv.grpnet(x, ...)
    \#\# Default S3 method:
    cv.grpnet (x,
        \(y\),
        group,
        weights = NULL,
        offset = NULL,
        alpha \(=c(0.01,0.25,0.5,0.75,1)\),
        gamma \(=c(3,4,5)\),
        type.measure \(=\) NULL,
        nfolds = 10,
        foldid = NULL,
        same.lambda = FALSE,
        parallel = FALSE,
        cluster = NULL,
        verbose = interactive(),
        ...)
    \#\# S3 method for class 'formula'
    cv.grpnet(formula,
            data,
            use.rk = TRUE,
            weights = NULL,
            offset = NULL,
            alpha \(=c(0.01,0.25,0.5,0.75,1)\),
            gamma \(=c(3,4,5)\),
            type.measure \(=\) NULL,
            nfolds = 10,
            foldid = NULL,
            same.lambda = FALSE,
            parallel = FALSE,
            cluster = NULL,
            verbose = interactive(),
            ...)
```


## Arguments

x
y
group Group label vector (factor, character, or integer) of length $p$. Predictors with the same label are grouped together for regularization.
formula Model formula: a symbolic description of the model to be fitted. Uses the same syntax as lm and glm.
data
Model (design) matrix of dimension nobs by nvars ( $n \times p$ ).
Response vector of length $n$. Matrix inputs are allowed for binomial and multinomial families (see "Binomial and multinomial" section in grpnet). Optional data frame containing the variables referenced in formula.

| use.rk | If TRUE (default), the rk.model.matrix function is used to build the model ma- <br> trix. Otherwise, the model.matrix function is used to build the model matrix. <br> Additional arguments to the rk. model. matrix function can be passed via the |
| :--- | :--- |
| _ . argument. |  |
| Optional vector of length $n$ with non-negative weights to use for weighted (pe- |  |
| nalized) likelihood estimation. Defaults to a vector of ones. |  |
| Optional vector of length $n$ with an a priori known term to be included in the |  |
| offset | model's linear predictor. Defaults to a vector of zeros. |
| alpha | Scalar or vector specifying the elastic net tuning parameter $\alpha$. If alpha is a vec- <br> tor (default), then (a) the same foldid is used to compute the cross-validation <br> error for each $\alpha$, and (b) the solution for the optimal $\alpha$ is returned. |
| gamma | Scalar or vector specifying the penalty hyperparameter $\gamma$ for MCP or SCAD. <br> If gamma is a vector (default), then (a) the same foldid is used to compute <br> the cross-validation error for each $\gamma$, and (b) the solution for the optimal $\gamma$ is <br> returned. |
| type.measure | Loss function for cross-validation. Options include: "deviance" for model <br> deviance, "mse" for mean squared error, "mae" for mean absolute error, or |
| "class" for classification error. Note that "class" is only available for bino- |  |

## Details

This function calls the grpnet function nfolds+1 times: once on the full dataset to obtain the lambda sequence, and once holding out each fold's data to evaluate the prediction error. The syntax
of (the default S3 method for) this function closely mimics that of the cv.glmnet function in the glmnet package (Friedman, Hastie, \& Tibshirani, 2010).
Let $\mathbf{D}_{u}=\left\{\mathbf{y}_{u}, \mathbf{X}_{u}\right\}$ denote the $u$-th fold's data, let $\mathbf{D}_{[u]}=\left\{\mathbf{y}_{[u]}, \mathbf{X}_{[u]}\right\}$ denote the full dataset excluding the $u$-th fold's data, and let $\boldsymbol{\beta}_{\lambda[u]}$ denote the coefficient estimates obtained from fitting the model to $\mathbf{D}_{[u]}$ using the regularization parameter $\lambda$.
The cross-validation error for the $u$-th fold is defined as

$$
E_{u}(\lambda)=C\left(\boldsymbol{\beta}_{\lambda[u]}, \mathbf{D}_{u}\right)
$$

where $C(\cdot, \cdot)$ denotes the cross-validation loss function that is specified by type.measure. For example, the "mse" loss function is defined as

$$
C\left(\boldsymbol{\beta}_{\lambda[u]}, \mathbf{D}_{u}\right)=\left\|\mathbf{y}_{u}-\mathbf{X}_{u} \boldsymbol{\beta}_{\lambda[u]}\right\|^{2}
$$

where $\|\cdot\|$ denotes the L2 norm.
The mean cross-validation error cvm is defined as

$$
\bar{E}(\lambda)=\frac{1}{v} \sum_{u=1}^{v} E_{u}(\lambda)
$$

where $v$ is the total number of folds. The standard error cvsd is defined as

$$
S(\lambda)=\sqrt{\frac{1}{v(v-1)} \sum_{u=1}^{v}\left(E_{u}(\lambda)-\bar{E}(\lambda)\right)^{2}}
$$

which is the classic definition of the standard error of the mean.

## Value

| lambda | regularization parameter sequence for the full data |
| :--- | :--- |
| cvm | mean cross-validation error for each lambda |
| cvsd | estimated standard error of cvm |
| cvup | upper curve: cvm + cvsd |
| cvlo | lower curve: cvm - cvsd |
| nzero | number of non-zero groups for each lambda |
| grpnet.fit | fitted grpnet object for the full data |
| lambda.min | value of lambda that minimizes cvm |
| lambda.1se | largest lambda such that cvm is within one cvsd from the minimum (see Note) |
| index | two-element vector giving the indices of lambda.min and lambda. 1se in the |
| lype.measure | lambda vector, i.e., c(minid, se1id) as defined in the Note <br> losction for cross-validation (used for plot label) |
| call | matched call |
| time | runtime in seconds to perform k-fold CV tuning <br> tune |
|  | data frame containing the tuning results, i.e., min(cvm) for each combo of alpha <br> and/or gamma |

## Note

lambda. 1 se is defined as follows:
minid<- which.min(cvm)
min1se <- cvm[minid] + cvsd[minid]
se1id<- which(cvm<= min1se)[1]
lambda. $1 \mathrm{se}<-$ lambda[se1id]

## Author(s)

Nathaniel E. Helwig [helwig@umn.edu](mailto:helwig@umn.edu)

## References

Breheny, P., \& Huang, J. (2015). Group descent algorithms for nonconvex penalized linear and logistic regression models with grouped predictors. Statistics and Computing, 25, 173-187. doi:10.1007/ s1122201394242
Friedman, J., Hastie, T., \& Tibshirani, R. (2010). Regularization paths for generalized linear models via coordinate descent. Journal of Statistical Software, 33(1), 1-22. doi:10.18637/jss.v033.i01
Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. Journal of Computational and Graphical Statistics. doi:10.1080/ 10618600.2024.2362232

Yang, Y., \& Zou, H. (2015). A fast unified algorithm for solving group-lasso penalize learning problems. Statistics and Computing, 25, 1129-1141. doi:10.1007/s1122201494985

## See Also

plot.cv.grpnet for plotting the cross-validation error curve predict.cv.grpnet for predicting from cv.grpnet objects grpnet for fitting group elastic net regularization paths

## Examples

```
######****###### family = "gaussian" ######****######
# load data
data(auto)
# 10-fold cv (formula method, response = mpg)
set.seed(1)
mod <- cv.grpnet(mpg ~ ., data = auto)
# print min and 1se solution info
mod
# plot cv error curve
plot(mod)
```

```
######***###### family = "binomial" ######***######
# load data
data(auto)
# redefine origin (Domestic vs Foreign)
auto$origin <- ifelse(auto$origin == "American", "Domestic", "Foreign")
# 10-fold cv (default method, response = origin with 2 levels)
set.seed(1)
mod <- cv.grpnet(origin ~ ., data = auto, family = "binomial")
# print min and 1se solution info
mod
# plot cv error curve
plot(mod)
######***###### family = "multinomial" ######***######
# load data
data(auto)
# 10-fold cv (formula method, response = origin with 3 levels)
set.seed(1)
mod <- cv.grpnet(origin ~ ., data = auto, family = "multinomial")
# print min and 1se solution info
mod
# plot cv error curve
plot(mod)
######***###### family = "poisson" ######***######
# load data
data(auto)
# 10-fold cv (formula method, response = horsepower)
set.seed(1)
mod <- cv.grpnet(horsepower ~ ., data = auto, family = "poisson")
# print min and 1se solution info
mod
# plot cv error curve
plot(mod)
```

```
######***###### family = "negative.binomial" ######***######
# load data
data(auto)
# 10-fold cv (formula method, response = horsepower)
set.seed(1)
mod <- cv.grpnet(horsepower ~ ., data = auto, family = "negative.binomial")
# print min and 1se solution info
mod
# plot cv error curve
plot(mod)
######***###### family = "Gamma" ######***######
# load data
data(auto)
# 10-fold cv (formula method, response = origin)
set.seed(1)
mod <- cv.grpnet(mpg ~ ., data = auto, family = "Gamma")
# print min and 1se solution info
mod
# plot cv error curve
plot(mod)
######***###### family = "inverse.gaussian" ######***######
# load data
data(auto)
# 10-fold cv (formula method, response = origin)
set.seed(1)
mod <- cv.grpnet(mpg ~ ., data = auto, family = "inverse.gaussian")
# print min and 1se solution info
mod
# plot cv error curve
plot(mod)
```

family.grpnet Prepare 'family' Argument for grpnet

## Description

Takes in the family argument from grpnet and returns a list containing the information needed for fitting and/or tuning the model.

## Usage

family.grpnet(object, theta $=1$ )

## Arguments

object one of the following characters specifying the exponential family: "gaussian", "binomial", "multinomial", "poisson", "negative.binomial", "Gamma", "inverse.gaussian"
theta Additional ("size") parameter for negative binomial responses, where the variance function is defined as $V(\mu)=\mu+\mu^{2} / \theta$

## Details

There is only one available link function for each family:

* gaussian (identity): $\mu=\mathbf{X}^{\top} \boldsymbol{\beta}$
* binomial (logit): $\log \left(\frac{\pi}{1-\pi}\right)=\mathbf{X}^{\top} \boldsymbol{\beta}$
* multinomial (symmetric): $\pi_{\ell}=\frac{\exp \left(\mathbf{X}^{\top} \boldsymbol{\beta}_{\ell}\right)}{\sum_{l=1}^{m} \exp \left(\mathbf{X}^{\top} \boldsymbol{\beta}_{l}\right)}$
* poisson $(\log ): \log (\mu)=\mathbf{X}^{\top} \boldsymbol{\beta}$
* negative.binomial $(\log ): \log (\mu)=\mathbf{X}^{\top} \boldsymbol{\beta}$
* Gamma (log): $\log (\mu)=\mathbf{X}^{\top} \boldsymbol{\beta}$
* inverse.gaussian $(\log ): \log (\mu)=\mathbf{X}^{\top} \boldsymbol{\beta}$


## Value

List with components:
family same as input object, i.e., character specifying the family
linkinv function for computing inverse of link function
dev.resids function for computing deviance residuals

## Note

For gaussian family, this returns the full output produced by gaussian.

## Author(s)

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## References

Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. Journal of Computational and Graphical Statistics. doi:10.1080/ 10618600.2024.2362232

## See Also

grpnet for fitting group elastic net regularization paths
cv.grpnet for k-fold cross-validation of lambda

## Examples

```
family.grpnet("gaussian")
family.grpnet("binomial")
family.grpnet("multinomial")
family.grpnet("poisson")
family.grpnet("negbin", theta = 10)
family.grpnet("Gamma")
family.grpnet("inverse.gaussian")
```

grpnet Fit a Group Elastic Net Regularized GLM/GAM

## Description

Fits generalized linear/additive models with a group elastic net penalty using an adaptively bounded gradient descent (ABGD) algorithm (Helwig, 2024). Predictor groups can be manually input (default S3 method) or inferred from the model (S3 "formula" method). The regularization path is computed at a data-generated (default) or user-provided sequence of lambda values.

## Usage

grpnet(x, ...)
\#\# Default S3 method:
grpnet (x,
$y$,
group,
family = c("gaussian", "binomial", "multinomial", "poisson",
"negative.binomial", "Gamma", "inverse.gaussian"),
weights = NULL,
offset $=$ NULL,

```
    alpha = 1,
    nlambda = 100,
    lambda.min.ratio = ifelse(nobs < nvars, 0.05, 0.0001),
    lambda = NULL,
    penalty.factor = NULL,
    penalty = c("LASSO", "MCP", "SCAD"),
    gamma = 4,
    theta = 1,
    standardized = !orthogonalized,
    orthogonalized = TRUE,
    intercept = TRUE,
    thresh = 1e-04,
    maxit = 1e05,
    proglang = c("Fortran", "R"),
    ...)
## S3 method for class 'formula'
grpnet(formula,
    data,
    use.rk = TRUE,
    family = c("gaussian", "binomial", "multinomial", "poisson",
            "negative.binomial", "Gamma", "inverse.gaussian"),
    weights = NULL,
    offset = NULL,
    alpha = 1,
    nlambda = 100,
    lambda.min.ratio = ifelse(nobs < nvars, 0.05, 0.0001),
    lambda = NULL,
    penalty.factor = NULL,
    penalty = c("LASSO", "MCP", "SCAD"),
    gamma = 4,
    theta = 1,
    standardized = !orthogonalized,
    orthogonalized = TRUE,
    thresh = 1e-04,
    maxit = 1e05,
    proglang = c("Fortran", "R"),
    ...)
```


## Arguments

x
y $\quad$ Response vector of length $n$. Matrix inputs are allowed for binomial and multinomial families (see "Binomial and multinomial" section).
group Group label vector (factor, character, or integer) of length $p$. Predictors with the same label are grouped together for regularization.
formula Model formula: a symbolic description of the model to be fitted. Uses the same syntax as $1 m$ and glm.
$\left.\begin{array}{ll}\text { data } & \text { Optional data frame containing the variables referenced in formula. } \\ \text { use.rk } & \text { If TRUE (default), the rk.model.matrix function is used to build the model ma- } \\ \text { trix. Otherwise, the model.matrix function is used to build the model matrix. } \\ & \text { Additional arguments to the rk.model.matrix function can be passed via the } \\ & \ldots \text { argument. } \\ \text { Character specifying the assumed distribution for the response variable. Par- } \\ \text { family } & \text { tial matching is allowed. Options include "gaussian" (real-valued response), } \\ \text { "binomial" (binary response), "multinomial" (multi-class response), "poisson" } \\ \text { (count response), "negative.binomial" (count response), "Gamma" (positive }\end{array}\right\}$
of whether standardization is used, the coefficients are always returned on the original data scale.
orthogonalized Logical indicating whether the predictors should be groupwise orthogonalized. If TRUE (default), each predictor group's design matrix is orthonormalized (i.e., $\mathbf{X}_{k}^{\top} \mathbf{X}_{k}=n \mathbf{I}_{k}$ ) before fitting the model. Regardless of whether orthogonalization is used, the coefficients are always returned on the original data scale.
intercept Logical indicating whether an intercept term should be included in the model. Note that the intercept is always unpenalized.
thresh Convergence threshold (tolerance). The algorithm is determined to have converged once the maximum relative change in the coefficients is below this threshold. See "Convergence" section.
maxit Maximum number of iterations to allow.
proglang Which programming language should be used to implement the ABGD algorithm? Options include "Fortran" (default) or "R".
... Additional arguments used by the default or formula method.

## Details

Consider a generalized linear model of the form

$$
g(\mu)=\mathbf{X}^{\top} \boldsymbol{\beta}
$$

where $\mu=E(Y \mid \mathbf{X})$ is the conditional expectation of the response $Y$ given the predictor vector $\mathbf{X}$, the function $g(\cdot)$ is a user-specified (invertible) link function, and $\boldsymbol{\beta}$ are the unknown regression coefficients. Furthermore, suppose that the predictors are grouped, such as

$$
\mathbf{X}^{\top} \boldsymbol{\beta}=\sum_{k=1}^{K} \mathbf{X}_{k}^{\top} \boldsymbol{\beta}_{k}
$$

where $\mathbf{X}=\left(\mathbf{X}_{1}, \ldots, \mathbf{X}_{K}\right)$ is the grouped predictor vector, and $\boldsymbol{\beta}=\left(\boldsymbol{\beta}_{1}, \ldots, \boldsymbol{\beta}_{K}\right)$ is the grouped coefficient vector.

Given $n$ observations, this function finds the $\boldsymbol{\beta}$ that minimizes

$$
L(\boldsymbol{\beta} \mid \mathbf{D})+\lambda P_{\alpha}(\boldsymbol{\beta})
$$

where $L(\boldsymbol{\beta} \mid \mathbf{D})$ is the loss function with $\mathbf{D}=\{\mathbf{y}, \mathbf{X}\}$ denoting the observed data, $P_{\alpha}(\boldsymbol{\beta})$ is the group elastic net penalty, and $\lambda \geq 0$ is the regularization parameter.
The loss function has the form

$$
L(\boldsymbol{\beta} \mid \mathbf{D})=\frac{1}{n} \sum_{i=1}^{n} w_{i} \ell_{i}\left(\boldsymbol{\beta} \mid \mathbf{D}_{i}\right)
$$

where $w_{i}>0$ are the user-supplied weights, and $\ell_{i}\left(\boldsymbol{\beta} \mid \mathbf{D}_{i}\right)$ is the $i$-th observation's contribution to the loss function. Note that $\ell(\cdot)=-\log \left(f_{Y}(\cdot)\right)$ denotes the negative log-likelihood function for the given family.
The group elastic net penalty function has the form

$$
P_{\alpha}(\boldsymbol{\beta})=\alpha P_{1}(\boldsymbol{\beta})+(1-\alpha) P_{2}(\boldsymbol{\beta})
$$

where $\alpha \in[0,1]$ is the user-specified alpha value,

$$
P_{1}(\boldsymbol{\beta})=\sum_{k=1}^{K} \omega_{k}\left\|\boldsymbol{\beta}_{k}\right\|
$$

is the group lasso penalty with $\omega_{k} \geq 0$ denoting the $k$-th group's penalty.factor, and

$$
P_{2}(\boldsymbol{\beta})=\frac{1}{2} \sum_{k=1}^{K} \omega_{k}\left\|\boldsymbol{\beta}_{k}\right\|^{2}
$$

is the group ridge penalty. Note that $\left\|\boldsymbol{\beta}_{k}\right\|^{2}=\boldsymbol{\beta}_{k}^{\top} \boldsymbol{\beta}_{k}$ denotes the squared Euclidean norm. When penalty \%in\% c ("MCP", "SCAD"), the group L1 penalty $P_{1}(\boldsymbol{\beta})$ is replaced by the group MCP or group SCAD penalty.

## Value

An object of class "grpnet" with the following elements:

| call | matched call |
| :--- | :--- |
| a0 | intercept sequence of length nlambda |
| beta | coefficient matrix of dimension nvars by nlambda |
| alpha | balance between the group L1 (lasso) and group L2 (ridge) penalty |
| lambda | sequence of regularization parameter values |
| family | exponential family defining the loss function |
| dev.ratio | proportion of (null) deviance explained for each lambda (= 1-dev / nulldev) |
| nulldev | null deviance for each lambda |
| df | effective degrees of freedom for each lambda |
| nzgrp | number of non-zero groups for each lambda |
| nzcoef | number of non-zero coefficients for each lambda |
| xsd | standard deviation of x for each group |
| ylev | levels of response variable (only for binomial and multinomial families) |
| nobs | number of observations |
| group | group label vector |
| ngroups | number of groups $K$ |
| npasses | number of iterations for each lambda |
| time | runtime in seconds to compute regularization path |
| offset | logical indicating if an offset was included |
| args | list of input argument values |
| formula | input formula (possibly after expansion) |
| term.labels | terms that appear in formula (if applicable) |
| rk.args | arguments for rk.model.matrix function (if applicable) |

## S3 'formula' method

Important: When using the S3 "formula" method, the S3 "predict" method forms the model matrix for the predictions by applying the model formula to the new data. As a result, to ensure that the corresponding S3 "predict" method works correctly, some formulaic features should be avoided.

Polynomials: When including polynomial terms, the poly function should be used with option raw $=$ TRUE. Default use of the poly function (with raw = FALSE) will work for fitting the model, but will result in invalid predictions for new data. Polynomials can also be included via the I function, but this isn't recommended because the polynomials terms wouldn't be grouped together, i.e., the terms $x$ and $I\left(x^{\wedge} 2\right)$ would be treated as two separate groups of size one instead of a single group of size two.

Splines: B-splines (and other spline bases) can be included via the S3 "formula" method. However, to ensure reasonable predictions for new data, it is necessary to specify the knots directly. For example, if $x$ is a vector with entries between zero and one, the code $b s(x, d f=5)$ will *not* produce valid predictions for new data, but the code $b s(x$, knots $=c(0.25,0.5,0.75)$, Boundary.knots $=c(0,1)$ ) will work as intended. Instead of attempting to integrate a call to bs() or rk () into the model formula, it is recommended that splines be included via the use. $\mathrm{rk}=$ TRUE argument.

## Family argument and link functions

Unlike the glm function, the family argument of the grpnet function

* should be a character vector (not a family object)
* does not allow for specification of a link function

Currently, there is only one available link function for each family:

* gaussian (identity): $\mu=\mathbf{X}^{\top} \boldsymbol{\beta}$
* binomial (logit): $\log \left(\frac{\pi}{1-\pi}\right)=\mathbf{X}^{\top} \boldsymbol{\beta}$
* multinomial (symmetric): $\pi_{\ell}=\frac{\exp \left(\mathbf{X}^{\top} \boldsymbol{\beta}_{\ell}\right)}{\sum_{l=1}^{m} \exp \left(\mathbf{X}^{\top} \boldsymbol{\beta}_{l}\right)}$
* poisson $(\log ): \log (\mu)=\mathbf{X}^{\top} \boldsymbol{\beta}$
* negative binomial $(\log ): \log (\mu)=\mathbf{X}^{\top} \boldsymbol{\beta}$
* Gamma (log): $\log (\mu)=\mathbf{X}^{\top} \boldsymbol{\beta}$
* inverse.gaussian $(\log ): \log (\mu)=\mathbf{X}^{\top} \boldsymbol{\beta}$


## Binomial and multinomial

For "binomial" responses, three different possibilities exist for the input response:

1. vector coercible into a factor with two levels
2. matrix with two columns (\# successes, \# failures)
3. numeric vector with entries between 0 and 1

In this case, the weights argument should be used specify the total number of trials.
For "multinomial" responses, two different possibilities exist for the input reponse:

1. vector coercible into a factor with more than two levels
2. matrix of integers (counts) for each category level

## Convergence

The algorithm is determined to have converged once
$\max _{j} \frac{\left|\beta_{j}-\beta_{j}^{\text {old }}\right|}{1+\left|\beta_{j}^{\text {old }}\right|}<\epsilon$
where $j \in\{1, \ldots, p\}$ and $\epsilon$ is the thresh argument.

## Note

The syntax of (the default S 3 method for) this function closely mimics that of the glmnet function in the glmnet package (Friedman, Hastie, \& Tibshirani, 2010).

## Author(s)

Nathaniel E. Helwig [helwig@umn.edu](mailto:helwig@umn.edu)

## References

Breheny, P., \& Huang, J. (2015). Group descent algorithms for nonconvex penalized linear and logistic regression models with grouped predictors. Statistics and Computing, 25, 173-187. doi:10.1007/ s1122201394242
Friedman, J., Hastie, T., \& Tibshirani, R. (2010). Regularization paths for generalized linear models via coordinate descent. Journal of Statistical Software, 33(1), 1-22. doi:10.18637/jss.v033.i01
Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. Journal of Computational and Graphical Statistics. doi:10.1080/ 10618600.2024.2362232

Yang, Y., \& Zou, H. (2015). A fast unified algorithm for solving group-lasso penalize learning problems. Statistics and Computing, 25, 1129-1141. doi:10.1007/s1122201494985

## See Also

plot.grpnet for plotting the regularization path
predict.grpnet for predicting from grpnet objects
cv.grpnet for k-fold cross-validation of lambda

## Examples

```
######****###### family = "gaussian" ######***######
# load data
data(auto)
# fit model (formula method, response = mpg)
mod <- grpnet(mpg ~ ., data = auto)
# print regularization path info
mod
# plot coefficient paths
```

```
plot(mod)
######***###### family = "binomial" ######***######
# load data
data(auto)
# redefine origin (Domestic vs Foreign)
auto$origin <- ifelse(auto$origin == "American", "Domestic", "Foreign")
# fit model (formula method, response = origin with 2 levels)
mod <- grpnet(origin ~ ., data = auto, family = "binomial")
# print regularization path info
mod
# plot coefficient paths
plot(mod)
######***###### family = "multinomial" ######****#####
# load data
data(auto)
# fit model (formula method, response = origin with 3 levels)
mod <- grpnet(origin ~ ., data = auto, family = "multinomial")
# print regularization path info
mod
# plot coefficient paths
plot(mod)
######***###### family = "poisson" ######***######
# load data
data(auto)
# fit model (formula method, response = horsepower)
mod <- grpnet(horsepower ~ ., data = auto, family = "poisson")
# print regularization path info
mod
# plot coefficient paths
plot(mod)
```

```
######***###### family = "negative.binomial" ######***######
# load data
data(auto)
# fit model (formula method, response = horsepower)
mod <- grpnet(horsepower ~ ., data = auto, family = "negative.binomial")
# print regularization path info
mod
# plot coefficient paths
plot(mod)
######***###### family = "Gamma" ######****######
# load data
data(auto)
# fit model (formula method, response = mpg)
mod <- grpnet(mpg ~ ., data = auto, family = "Gamma")
# print regularization path info
mod
# plot coefficient paths
plot(mod)
######***###### family = "inverse.gaussian" ######***######
# load data
data(auto)
# fit model (formula method, response = mpg)
mod <- grpnet(mpg ~ ., data = auto, family = "inverse.gaussian")
# print regularization path info
mod
# plot coefficient paths
plot(mod)
```


## Description

Plots the mean cross-validation error, along with lower and upper standard deviation curves, as a function of $\log$ (lambda).

## Usage

```
## S3 method for class 'cv.grpnet'
plot(x, sign.lambda = 1, nzero = TRUE, ...)
```


## Arguments

x
sign. lambda Default plots $\log (l a m b d a)$ on the $x$-axis. Set to -1 to plot $-1 * \log$ (lambda) on the x -axis instead.
nzero $\quad$ Should the number of non-zero groups be printed on the top of the $x$-axis?
... Additional arguments passed to the plot function.

## Details

Produces cross-validation plot only (i.e., nothing is returned).

## Value

No return value (produces a plot)

## Note

Syntax and functionality were modeled after the plot.cv.glmnet function in the glmnet package (Friedman, Hastie, \& Tibshirani, 2010).

## Author(s)

Nathaniel E. Helwig [helwig@umn.edu](mailto:helwig@umn.edu)

## References

Friedman, J., Hastie, T., \& Tibshirani, R. (2010). Regularization paths for generalized linear models via coordinate descent. Journal of Statistical Software, 33(1), 1-22. doi:10.18637/jss.v033.i01
Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. Journal of Computational and Graphical Statistics. doi:10.1080/ 10618600.2024.2362232

## See Also

cv.grpnet for k-fold cross-validation of lambda
plot.grpnet for plotting the regularization path

## Examples

```
# see 'cv.grpnet' for plotting examples
?cv.grpnet
```

plot.grpnet Plot Regularization Path for grpnet Fits

## Description

Creates a profile plot of the reguarlization paths for a fit group elastic net regularized GLM (grpnet) object.

## Usage

```
## S3 method for class 'grpnet'
plot(x, type = c("coef", "imp", "norm", "znorm"),
    newx, newdata, intercept = FALSE,
    color.by.group = TRUE, col = NULL, ...)
```


## Arguments

$x \quad$ Object of class "grpnet"
type What to plot on the Y-axis: "coef" for coefficient values, "imp" for importance of each group's contribution, "norm" for L2 norm of coefficients for each group, or "znorm" for L2 norm of standardized coefficients for each group.
newx Matrix of new $x$ scores for prediction (default S3 method). Ignored unless type = "imp".
newdata Data frame of new data scores for prediction (S3 "formula" method). Ignored unless type = "imp".
intercept Should the intercept be included in the plot?
color.by.group If TRUE (default), the coefficient paths are colored according to their group membership using the colors in col. If FALSE, all coefficient paths are plotted the same color.
col If color.by.group = TRUE, this should be a vector of length $K$ giving a color label for each group. If color. by.group = FASLE, this should be a character specifying a single (common) color. Default of col = NULL is the same as col = 1:K or col = "black".
... Additional arguments passed to the plot function.

## Details

Syntax and functionality were modeled after the plot.glmnet function in the glmnet package (Friedman, Hastie, \& Tibshirani, 2010).

## Value

Produces a profile plot showing the requested type (y-axis) as a function of $\log$ (lambda) (x-axis).

## Note

If x is a multinomial model, the coefficients for each response class are plotted in a separate plot.

## Author(s)

Nathaniel E. Helwig [helwig@umn.edu](mailto:helwig@umn.edu)

## References

Friedman, J., Hastie, T., \& Tibshirani, R. (2010). Regularization paths for generalized linear models via coordinate descent. Journal of Statistical Software, 33(1), 1-22. doi:10.18637/jss.v033.i01
Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. Journal of Computational and Graphical Statistics. doi:10.1080/ 10618600.2024.2362232

## See Also

grpnet for fitting grpnet regularization paths
plot.cv.grpnet for plotting cv.grpnet objects

## Examples

```
# see 'grpnet' for plotting examples
?grpnet
```

```
predict.cv.grpnet Predict Method for cv.grpnet Fits
```


## Description

Obtain predictions from a cross-validated group elastic net regularized GLM (cv.grpnet) object.

## Usage

```
## S3 method for class 'cv.grpnet'
predict(object,
            newx,
            newdata,
            s = c("lambda.1se", "lambda.min"),
            type = c("link", "response", "class", "terms",
                "importance", "coefficients", "nonzero", "groups",
                "ncoefs", "ngroups", "norm", "znorm"),
    ...)
```


## Arguments

object Object of class "cv.grpnet"
newx Matrix of new x scores for prediction (default S3 method). Must have $p$ columns arranged in the same order as the $x$ matrix used to fit the model.
newdata Data frame of new data scores for prediction (S3 "formula" method). Must contain all variables in the formula used to fit the model.
s
Lambda value(s) at which predictions should be obtained. Can input a character ("lambda.min" or "lambda.1se") or a numeric vector. Default of "lambda.min" uses the lambda value that minimizes the mean cross-validated error.
type Type of prediction to return. "link" gives predictions on the link scale ( $\eta$ ). "response" gives predictions on the mean scale ( $\mu$ ). "class" gives predicted class labels (for "binomial" and "multinomial" families). "terms" gives the predictions for each term (group) in the model $\left(\eta_{k}\right)$. "importance" gives the variable importance index for each term (group) in the model. "coefficients" returns the coefficients used for predictions. "nonzero" returns a list giving the indices of non-zero coefficients for each $s$. "groups" returns a list giving the labels of nonzero groups for each s. "ncoefs" returns the number of non-zero coefficients for each s. "ngroups" returns the number of non-zero groups for each s. "norm" returns the L2 norm of each group's (raw) coefficients for each s. "znorm" returns the L2 norm of each group's standardized coefficients for each s.
... Additional arguments (ignored)

## Details

Predictions are calculated from the grpnet object fit to the full sample of data, which is stored as object\$grpnet.fit

See predict.grpnet for further details on the calculation of the different types of predictions.

## Value

Depends on three factors...

1. the exponential family distribution
2. the length of the input $s$
3. the type of prediction requested

See predict.grpnet for details

## Note

Syntax is inspired by the predict.cv.glmnet function in the glmnet package (Friedman, Hastie, \& Tibshirani, 2010).

## Author(s)

Nathaniel E. Helwig [helwig@umn.edu](mailto:helwig@umn.edu)

## References

Friedman, J., Hastie, T., \& Tibshirani, R. (2010). Regularization paths for generalized linear models via coordinate descent. Journal of Statistical Software, 33(1), 1-22. doi:10.18637/jss.v033.i01
Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. Journal of Computational and Graphical Statistics. doi:10.1080/ 10618600.2024.2362232

## See Also

cv.grpnet for k-fold cross-validation of lambda predict.grpnet for predicting from grpnet objects

## Examples

```
######****###### family = "gaussian" ######***######
# load data
data(auto)
# 10-fold cv (formula method, response = mpg)
set.seed(1)
mod <- cv.grpnet(mpg ~ ., data = auto)
# get fitted values at "lambda.1se"
fit.1se <- predict(mod, newdata = auto)
# get fitted values at "lambda.min"
fit.min <- predict(mod, newdata = auto, s = "lambda.min")
# compare mean absolute error for two solutions
mean(abs(auto$mpg - fit.1se))
mean(abs(auto$mpg - fit.min))
######***###### family = "binomial" ######***######
# load data
data(auto)
# redefine origin (Domestic vs Foreign)
auto$origin <- ifelse(auto$origin == "American", "Domestic", "Foreign")
# 10-fold cv (default method, response = origin with 2 levels)
set.seed(1)
mod <- cv.grpnet(origin ~ ., data = auto, family = "binomial")
# get predicted classes at "lambda.1se"
fit.1se <- predict(mod, newdata = auto, type = "class")
# get predicted classes at "lambda.min"
```

```
fit.min <- predict(mod, newdata = auto, type = "class", s = "lambda.min")
# compare misclassification rate for two solutions
1 - mean(auto$origin == fit.1se)
1 - mean(auto$origin == fit.min)
######***###### family = "multinomial" ######***######
# load data
data(auto)
# 10-fold cv (formula method, response = origin with 3 levels)
set.seed(1)
mod <- cv.grpnet(origin ~ ., data = auto, family = "multinomial")
# get predicted classes at "lambda.1se"
fit.1se <- predict(mod, newdata = auto, type = "class")
# get predicted classes at "lambda.min"
fit.min <- predict(mod, newdata = auto, type = "class", s = "lambda.min")
# compare misclassification rate for two solutions
1 - mean(auto$origin == fit.1se)
1 - mean(auto$origin == fit.min)
######***###### family = "poisson" ######***#######
# load data
data(auto)
# 10-fold cv (formula method, response = horsepower)
set.seed(1)
mod <- cv.grpnet(horsepower ~ ., data = auto, family = "poisson")
# get fitted values at "lambda.1se"
fit.1se <- predict(mod, newdata = auto, type = "response")
# get fitted values at "lambda.min"
fit.min <- predict(mod, newdata = auto, type = "response", s = "lambda.min")
# compare mean absolute error for two solutions
mean(abs(auto$horsepower - fit.1se))
mean(abs(auto$horsepower - fit.min))
######***###### family = "negative.binomial" ######***######
# load data
```

```
data(auto)
# 10-fold cv (formula method, response = horsepower)
set.seed(1)
mod <- cv.grpnet(horsepower ~ ., data = auto, family = "negative.binomial")
# get fitted values at "lambda.1se"
fit.1se <- predict(mod, newdata = auto, type = "response")
# get fitted values at "lambda.min"
fit.min <- predict(mod, newdata = auto, type = "response", s = "lambda.min")
# compare mean absolute error for two solutions
mean(abs(auto$horsepower - fit.1se))
mean(abs(auto$horsepower - fit.min))
######***###### family = "Gamma" ######***######
# load data
data(auto)
# 10-fold cv (formula method, response = origin)
set.seed(1)
mod <- cv.grpnet(mpg ~ ., data = auto, family = "Gamma")
# get fitted values at "lambda.1se"
fit.1se <- predict(mod, newdata = auto, type = "response")
# get fitted values at "lambda.min"
fit.min <- predict(mod, newdata = auto, type = "response", s = "lambda.min")
# compare mean absolute error for two solutions
mean(abs(auto$mpg - fit.1se))
mean(abs(auto$mpg - fit.min))
######***###### family = "inverse.gaussian" ######***######
# load data
data(auto)
# 10-fold cv (formula method, response = origin)
set.seed(1)
mod <- cv.grpnet(mpg ~ ., data = auto, family = "inverse.gaussian")
# get fitted values at "lambda.1se"
fit.1se <- predict(mod, newdata = auto, type = "response")
# get fitted values at "lambda.min"
fit.min <- predict(mod, newdata = auto, type = "response", s = "lambda.min")
```

```
# compare mean absolute error for two solutions
mean(abs(auto$mpg - fit.1se))
mean(abs(auto$mpg - fit.min))
```

```
predict.grpnet Predict Method for grpnet Fits
```


## Description

Obtain predictions from a fit group elastic net regularized GLM (grpnet) object.

## Usage

```
## S3 method for class 'grpnet'
predict(object,
    newx,
    newdata,
    s = NULL,
    type = c("link", "response", "class", "terms",
                            "importance", "coefficients", "nonzero", "groups",
                            "ncoefs", "ngroups", "norm", "znorm"),
    ...)
```


## Arguments

object Object of class "grpnet"
newx Matrix of new x scores for prediction (default S3 method). Must have $p$ columns arranged in the same order as the $x$ matrix used to fit the model. Ignored for the last six types of predictions.
newdata Data frame of new data scores for prediction (S3 "formula" method). Must contain all variables in the formula used to fit the model. Ignored for the last six types of predictions.
s
Lambda value(s) at which predictions should be obtained. Default uses $\mathrm{s}=$ object\$lambda. Interpolation is used for $s$ values that are not included in object\$lambda.
type Type of prediction to return. "link" gives predictions on the link scale $(\eta)$. "response" gives predictions on the mean scale ( $\mu$ ). "class" gives predicted class labels (for "binomial" and "multinomial" families). "terms" gives the predictions for each term (group) in the model $\left(\eta_{k}\right)$. "importance" gives the variable importance index for each term (group) in the model. "coefficients" returns the coefficients used for predictions. "nonzero" returns a list giving the indices of non-zero coefficients for each s. "groups" returns a list giving the labels of nonzero groups for each s. "ncoefs" returns the number of non-zero coefficients for
each s. "ngroups" returns the number of non-zero groups for each s. "norm" returns the L2 norm of each group's (raw) coefficients for each s. "znorm" returns the L2 norm of each group's standardized coefficients for each s.
... Additional arguments (ignored)

## Details

When type == "link", the predictions for each $\lambda$ have the form

$$
\boldsymbol{\eta}_{\lambda}=\mathbf{X}_{\text {new }} \boldsymbol{\beta}_{\lambda}
$$

where $\mathbf{X}_{\text {new }}$ is the argument newx (or the design matrix created from newdata by applying object\$formula) and $\boldsymbol{\beta}_{\lambda}$ is the coefficient vector corresponding to $\lambda$.
When type == "response", the predictions for each $\lambda$ have the form

$$
\boldsymbol{\mu}_{\lambda}=g^{-1}\left(\boldsymbol{\eta}_{\lambda}\right)
$$

where $g^{-1}(\cdot)$ is the inverse link function stored in object\$family\$linkinv.
When type == "class", the predictions for each $\lambda$ have the form

$$
\mathbf{y}_{\lambda}=\arg \max _{l} \boldsymbol{\mu}_{\lambda}(l)
$$

where $\boldsymbol{\mu}_{\lambda}(l)$ gives the predicted probability that each observation belongs to the $l$-th category (for $l=1, \ldots, m$ ) using the regularization parameter $\lambda$.
When type == "terms", the groupwise predictions for each $\lambda$ have the form

$$
\boldsymbol{\eta}_{k \lambda}=\mathbf{X}_{k}^{(\text {new })} \boldsymbol{\beta}_{k \lambda}
$$

where $\mathbf{X}_{k}^{(\text {new })}$ is the portion of the argument newx (or the design matrix created from newdata by applying object\$formula) that corresponds to the $k$-th term/group, and $\boldsymbol{\beta}_{k \lambda}$ are the corresponding coefficients.
When type == "importance", the variable importance indices are defined as

$$
\pi_{k}=\left(\boldsymbol{\eta}_{k \lambda}^{\top} \mathbf{C} \boldsymbol{\eta}_{0 \lambda}\right)\left(\boldsymbol{\eta}_{0 \lambda}^{\top} \mathbf{C} \boldsymbol{\eta}_{0 \lambda}\right)^{-1}
$$

where $\mathbf{C}=\left(\mathbf{I}_{n}-\frac{1}{n} \mathbf{1}_{n} \mathbf{1}_{n}^{\top}\right)$ denotes a centering matrix, and $\boldsymbol{\eta}_{0 \lambda}=\sum_{k=1}^{K} \boldsymbol{\eta}_{k \lambda}$. Note that $\sum_{k=1}^{K} \pi_{k}=$ 1 , but some $\pi_{k}$ could be negative. When they are positive, $\pi_{k}$ gives the approximate proportion of model (explained) variation that is attributed to the $k$-th term.

## Value

Depends on three factors...

1. the exponential family distribution
2. the length of the input s
3. the type of prediction requested

For most response variables, the typical output will be...
*
a matrix of dimension $c$ (newnobs, length $(s)$ ) if length $(s)>1$

* a vector of length newnobs if length(s) ==1

For multinomial response variables, the typical output will be...

* an array of dimension c(newnobs, length(object\$ylev), length(s)) if type \%in\% c("link", "response")
* a matrix of dimension c(newobs, length(s)) if type == "class"

Note: if type == "class", then the output will be the same class as object\$ylev. Otherwise, the output will be real-valued (or integer for the counts).

If type == "terms" and family != "multinomial", the output will be...

* an array of dimension $c$ (newnobs, nterms, length(s)) if length $(s)>1$
* a matrix of dimension $c$ (newnobs, nterms) if length(s) $==1$

If type == "terms" and family == "multinomial", the output will be a list of length length (object\$ylev) where each element gives the terms for the corresponding response class.

If type == "importance" and family != "multinomial", the output will be...

* a matrix of dimension $c$ (nterms, length(s)) if length (s) >1
* a vector of length nterms if length(s)==1

If type == "importance" and family == "multinomial", the output will be a list of length length(object\$ylev) where each element gives the importance for the corresponding response class. If length $(s)==1$, the output will be simplified to matrix.

If type == "coefficients", the output will be the same as that produced by coef.grpnet.
If type == "nonzero", the output will be a list of length length(s) where each element is a vector of integers (indices).

If type == "groups", the output will be a list of length length(s) where each element is a vector of characters (term. labels).

If type \%in\% c("ncoefs", "ngroups"), the output will be a vector of length length(s) where each element is an integer.
If type == "norm", the output will be a matrix of dimension $c(K$, length $(s))$, where each cell gives the L 2 norm for the corresponding group and smoothing parameter. Note that K denotes the number of groups.

## Note

Some internal code (e.g., used for the interpolation) is borrowed from the predict.glmnet function in the glmnet package (Friedman, Hastie, \& Tibshirani, 2010).

## Author(s)

Nathaniel E. Helwig [helwig@umn.edu](mailto:helwig@umn.edu)

## References

Friedman, J., Hastie, T., \& Tibshirani, R. (2010). Regularization paths for generalized linear models via coordinate descent. Journal of Statistical Software, 33(1), 1-22. doi:10.18637/jss.v033.i01
Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. Journal of Computational and Graphical Statistics. doi:10.1080/ 10618600.2024.2362232

## See Also

grpnet for fitting grpnet regularization paths
predict.cv.grpnet for predicting from cv.grpnet objects

## Examples

```
######***###### family = "gaussian" ######***######
# load data
data(auto)
# fit model (formula method, response = mpg)
mod <- grpnet(mpg ~ ., data = auto)
# get fitted values for regularization path (output = 392 x 100 matrix)
fit.path <- predict(mod, newdata = auto)
# get fitted values at 3 particular points (output = 392 x 3 matrix)
fit.some <- predict(mod, newdata = auto, s = c(1.5, 1, 0.5))
# compare rmse for solutions
rmse.path <- sqrt(colMeans((auto$mpg - fit.path)^2))
rmse.some <- sqrt(colMeans((auto$mpg - fit.some)^2))
plot(log(mod$lambda), rmse.path, cex = 0.5)
points(log(c(1.5, 1, 0.5)), rmse.some, pch = 0, col = "red")
######***###### family = "binomial" ######***######
# load data
data(auto)
# redefine origin (Domestic vs Foreign)
auto$origin <- ifelse(auto$origin == "American", "Domestic", "Foreign")
# fit model (formula method, response = origin with 2 levels)
mod <- grpnet(origin ~ ., data = auto, family = "binomial")
# get predicted classes for regularization path (output = 392 x 100 matrix)
fit.path <- predict(mod, newdata = auto, type = "class")
# get predicted classes at 3 particular points (output = 392 x 3 matrix)
```

```
fit.some <- predict(mod, newdata = auto, type = "class", s = c(.15, .1, .05))
# compare misclassification rate for solutions
miss.path <- 1 - colMeans(auto$origin == fit.path)
miss.some <- 1 - colMeans(auto$origin == fit.some)
plot(log(mod$lambda), miss.path, cex = 0.5)
points(log(c(.15, .1, .05)), miss.some, pch = 0, col = "red")
######***###### family = "multinomial" ######***#######
# load data
data(auto)
# fit model (formula method, response = origin with 3 levels)
mod <- grpnet(origin ~ ., data = auto, family = "multinomial")
# get predicted classes for regularization path (output = 392 x 100 matrix)
fit.path <- predict(mod, newdata = auto, type = "class")
# get predicted classes at 3 particular points (output = 392 x 3 matrix)
fit.some <- predict(mod, newdata = auto, type = "class", s = c(.1, .01, .001))
# compare misclassification rate for solutions
miss.path <- 1 - colMeans(auto$origin == fit.path)
miss.some <- 1 - colMeans(auto$origin == fit.some)
plot(log(mod$lambda), miss.path, cex = 0.5)
points(log(c(.1, .01, .001)), miss.some, pch = 0, col = "red")
######***###### family = "poisson" ######***#######
# load data
data(auto)
# fit model (formula method, response = horsepower)
mod <- grpnet(horsepower ~ ., data = auto, family = "poisson")
# get fitted values for regularization path (output = 392 x 100 matrix)
fit.path <- predict(mod, newdata = auto, type = "response")
# get fitted values at 3 particular points (output = 392 x 3 matrix)
fit.some <- predict(mod, newdata = auto, type = "response", s = c(15, 10, 5))
# compare rmse for solutions
rmse.path <- sqrt(colMeans((auto$horsepower - fit.path)^2))
rmse.some <- sqrt(colMeans((auto$horsepower - fit.some)^2))
plot(log(mod$lambda), rmse.path, cex = 0.5)
points(log(c(15, 10, 5)), rmse.some, pch = 0, col = "red")
```

```
######***###### family = "negative.binomial" ######***######
# load data
data(auto)
# fit model (formula method, response = horsepower)
mod <- grpnet(horsepower ~ ., data = auto, family = "negative.binomial")
# get fitted values for regularization path (output = 392 x 100 matrix)
fit.path <- predict(mod, newdata = auto, type = "response")
# get fitted values at 3 particular points (output = 392 x 3 matrix)
fit.some <- predict(mod, newdata = auto, type = "response", s = c(0.1, 0.01, 0.001))
# compare rmse for solutions
rmse.path <- sqrt(colMeans((auto$horsepower - fit.path)^2))
rmse.some <- sqrt(colMeans((auto$horsepower - fit.some)^2))
plot(log(mod$lambda), rmse.path, cex = 0.5)
points(log(c(0.1, 0.01, 0.001)), rmse.some, pch = 0, col = "red")
######****##### family = "Gamma" ######****#####
# load data
data(auto)
# fit model (formula method, response = mpg)
mod <- grpnet(mpg ~ ., data = auto, family = "Gamma")
# get fitted values for regularization path (output = 392 x 100 matrix)
fit.path <- predict(mod, newdata = auto, type = "response")
# get fitted values at 3 particular points (output = 392 x 3 matrix)
fit.some <- predict(mod, newdata = auto, type = "response", s = c(0.1, 0.01, 0.001))
# compare rmse for solutions
rmse.path <- sqrt(colMeans((auto$mpg - fit.path)^2))
rmse.some <- sqrt(colMeans((auto$mpg - fit.some)^2))
plot(log(mod$lambda), rmse.path, cex = 0.5)
points(log(c(0.1, 0.01, 0.001)), rmse.some, pch = 0, col = "red")
######****##### family = "inverse.gaussian" ######***######
# load data
data(auto)
# fit model (formula method, response = mpg)
mod <- grpnet(mpg ~ ., data = auto, family = "inverse.gaussian")
```

```
# get fitted values for regularization path (output = 392 x 100 matrix)
fit.path <- predict(mod, newdata = auto, type = "response")
# get fitted values at 3 particular points (output = 392 x 3 matrix)
fit.some <- predict(mod, newdata = auto, type = "response", s = c(0.005, 0.001, 0.0001))
# compare rmse for solutions
rmse.path <- sqrt(colMeans((auto$mpg - fit.path)^2))
rmse.some <- sqrt(colMeans((auto$mpg - fit.some)^2))
plot(log(mod$lambda), rmse.path, cex = 0.5)
points(log(c(0.005, 0.001, 0.0001)), rmse.some, pch = 0, col = "red")
```

```
print S3 'print' Methods for grpnet
```


## Description

Prints some basic information about the coefficients (for coef.grpnet objects), observed crossvalidation error (for cv.grpnet objects), or the computed regularization path (for grpnet objects).

## Usage

```
## S3 method for class 'coef.grpnet'
print(x, ...)
## S3 method for class 'cv.grpnet'
print(x, digits = max(3, getOption("digits") - 3), ...)
## S3 method for class 'grpnet'
print(x, ...)
```


## Arguments

| $x$ | an object of class coef.grpnet, cv.grpnet, or grpnet |
| :--- | :--- |
| digits | the number of digits to print (must be a positive integer) |
| $\ldots$ | additional arguments for print (currently ignored) |

## Details

For coef.grpnet objects, prints the non-zero coefficients and uses "." for coefficients shrunk to zero.
For cv.grpnet objects, prints the function call, the cross-validation type.measure, and a two-row table with information about the min and 1 se solutions.

For grpnet objects, prints a data frame with columns

* nGrp: number of non-zero groups for each lambda
* Df: effective degrees of freedom for each lambda
* \%Dev: percentage of null deviance explained for each lambda
* Lambda: the values of lambda


## Value

No return value (produces a printout)

## Note

Some syntax and functionality were modeled after the print functions in the glmnet package (Friedman, Hastie, \& Tibshirani, 2010).

## Author(s)

Nathaniel E. Helwig [helwig@umn.edu](mailto:helwig@umn.edu)

## References

Friedman, J., Hastie, T., \& Tibshirani, R. (2010). Regularization paths for generalized linear models via coordinate descent. Journal of Statistical Software, 33(1), 1-22. doi:10.18637/jss.v033.i01

Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. Journal of Computational and Graphical Statistics. doi:10.1080/ 10618600.2024.2362232

## See Also

coef.grpnet for extracting coefficients
cv.grpnet for k-fold cross-validation of lambda
grpnet for fitting grpnet regularization paths

## Examples

```
# see 'coef.grpnet' for coefficient printing examples
?coef.grpnet
# see 'cv.grpnet' for cross-validation error printing examples
?cv.grpnet
# see 'grpnet' for regularization path printing examples
?grpnet
```

```
rk Reproducing Kernel Basis
```


## Description

Generate a reproducing kernel basis matrix for a nominal, ordinal, or polynomial smoothing spline.

## Usage

rk(x, df = NULL, knots = NULL, m = NULL, intercept = FALSE,
Boundary.knots $=$ NULL, warn.outside $=$ TRUE,
periodic = FALSE, xlev = levels(x))

## Arguments

x
df the degrees of freedom, i.e., number of knots to place at quantiles of x. Defaults to 5 but ignored if knots are provided.
knots the breakpoints (knots) defining the spline. If knots are provided, the df is defined as length(unique(c(knots, Boundary.knots))).
m
the derivative penalty order: $0=$ ordinal spline, $1=$ linear spline, $2=$ cubic spline, 3 = quintic spline
intercept should an intercept be included in the basis?
Boundary.knots the boundary points for spline basis. Defaults to range (x).
warn. outside if TRUE, a warning is provided when $x$ values are outside of the Boundary. knots
periodic should the spline basis functions be constrained to be periodic with respect to the Boundary.knots?
$\mathrm{xlev} \quad$ levels of x (only applicable if x is a factor)

## Details

Given a vector of function realizations $f$, suppose that $f=X \beta$, where $X$ is the (unregularized) spline basis and $\beta$ is the coefficient vector. Let $Q$ denote the postive semi-definite penalty matrix, such that $\beta^{\top} Q \beta$ defines the roughness penalty for the spline. See Helwig (2017) for the form of $X$ and $Q$ for the various types of splines.

Consider the spectral parameterization of the form $f=Z \alpha$ where

$$
Z=X Q^{-1 / 2}
$$

is the regularized spline basis (that is returned by this function), and $\alpha=Q^{1 / 2} \beta$ are the reparameterized coefficients. Note that $X \beta=Z \alpha$ and $\beta^{\top} Q \beta=\alpha^{\top} \alpha$, so the spectral parameterization absorbs the penalty into the coefficients (see Helwig, 2021, 2024).
Syntax of this function is designed to mimic the syntax of the bs function.

## Value

Returns a basis function matrix of dimension $n$ by $d f$ (plus 1 if an intercept is included) with the following attributes:

| df | degrees of freedom |
| :--- | :--- |
| knots | knots for spline basis |
| $m$ | derivative penalty order |
| intercept | was an intercept included? |
| Boundary.knots | boundary points of $x$ |
| periodic | is the basis periodic? |
| xlev | factor levels (if applicable) |

## Note

The (default) type of spline basis depends on the class of the input $x$ object:

* If $x$ is an unordered factor, then a nominal spline basis is used
* If $x$ is an ordered factor (and $m=$ NULL), then an ordinal spline basis is used
* If $x$ is an integer or numeric (and $m=N U L L$ ), then a cubic spline basis is used

Note that you can override the default behavior by specifying the $m$ argument.

## Author(s)

Nathaniel E. Helwig [helwig@umn.edu](mailto:helwig@umn.edu)

## References

Helwig, N. E. (2017). Regression with ordered predictors via ordinal smoothing splines. Frontiers in Applied Mathematics and Statistics, 3(15), 1-13. doi:10.3389/fams.2017.00015
Helwig, N. E. (2021). Spectrally sparse nonparametric regression via elastic net regularized smoothers. Journal of Computational and Graphical Statistics, 30(1), 182-191. doi:10.1080/10618600.2020.1806855

Helwig, N. E. (2024). Precise tensor product smoothing via spectral splines. Stats, 7(1), 34-53. doi:10.3390/stats7010003

## Examples

```
######***###### NOMINAL SPLINE BASIS ######***######
x <- as.factor(LETTERS[1:5])
basis <- rk(x)
plot(1:5, basis[,1], t = "l", ylim = extendrange(basis))
for(j in 2:ncol(basis)){
    lines(1:5, basis[,j], col = j)
}
```

x <- as.ordered(LETTERS[1:5])
basis <- rk(x)
plot(1:5, basis[,1], t = "l", ylim = extendrange(basis))
for(j in 2:ncol(basis)){
lines(1:5, basis[,j], col = j)
}
\#\#\#\#\#\#***\#\#\#\#\#\# LINEAR SPLINE BASIS \#\#\#\#\#\#***\#\#\#\#\#\#
x <- seq(0, 1, length.out = 101)
basis <- rk(x, m = 1)
plot(x, basis[,1], t = "l", ylim = extendrange(basis))
for(j in 2:ncol(basis)){
lines(x, basis[,j], col = j)
}
\#\#\#\#\#\#***\#\#\#\#\#\# CUBIC SPLINE BASIS \#\#\#\#\#\#****\#\#\#\#\#\#
x <- seq(0, 1, length.out = 101)
basis <- rk(x)
basis <- scale(basis) \# for visualization only!
plot(x, basis[,1], t = "l", ylim = extendrange(basis))
for(j in 2:ncol(basis)){
lines(x, basis[,j], col = j)
}
\#\#\#\#\#\#***\#\#\#\#\#\# QUINTIC SPLINE BASIS \#\#\#\#\#\#***\#\#\#\#\#\#
x <- seq(0, 1, length.out = 101)
basis <- rk(x, m = 3)
basis <- scale(basis) \# for visualization only!
plot(x, basis[,1], t = "l", ylim = extendrange(basis))
for(j in 2:ncol(basis)){
lines(x, basis[,j], col = j)
}

```
rk.model.matrix Construct Design Matrices via Reproducing Kernels

\section*{Description}

Creates a design (or model) matrix using the rk function to expand variables via a reproducing kernel basis.

\section*{Usage}
rk.model.matrix(object, data = environment(object), ...)

\section*{Arguments}
object a formula or terms object describing the fit model
data a data frame containing the variables referenced in object
... additional arguments passed to the rk function, e.g., df, knots, m, etc. Arguments must be passed as a named list, see Examples.

\section*{Details}

Designed to be a more flexible alternative to the model.matrix function. The rk function is used to construct a marginal basis for each variable that appears in the input object. Tensor product interactions are formed by taking a row. kronecker product of marginal basis matrices. Interactions of any order are supported using standard formulaic conventions, see Note.

\section*{Value}

The design matrix corresponding to the input formula and data, which has the following attributes:
\begin{tabular}{ll} 
assign & \begin{tabular}{l} 
an integer vector with an entry for each column in the matrix giving the term in \\
the formula which gave rise to the column
\end{tabular} \\
term. labels & a character vector containing the labels for each of the terms in the model \\
knots & a named list giving the knots used for each variable in the formula \\
\(m\) & a named list giving the penalty order used for each variable in the formula \\
periodic & a named list giving the periodicity used for each variable in the formula \\
xlev & a named list giving the factor levels used for each variable in the formula
\end{tabular}

\section*{Note}

For formulas of the form \(y \sim x+z\), the constructed model matrix has the form cbind (rk(x), \(r k(z)\) ), which simply concatenates the two marginal basis matrices. For formulas of the form \(y \sim x\) : z, the constructed model matrix has the form row. kronecker (rk(x), rk(z)), where row. kronecker denotes the row-wise kronecker product. The formula \(y \sim x * z\) is a shorthand for \(y \sim x+z+x: z\), which concatenates the two previous results. Unless it is suppressed (using 0+), the first column of the basis will be a column of ones named (Intercept).

\section*{Author(s)}

Nathaniel E. Helwig <helwig@umn.edu>

\section*{References}

Helwig, N. E. (2017). Regression with ordered predictors via ordinal smoothing splines. Frontiers in Applied Mathematics and Statistics, 3(15), 1-13. doi:10.3389/fams.2017.00015
Helwig, N. E. (2021). Spectrally sparse nonparametric regression via elastic net regularized smoothers. Journal of Computational and Graphical Statistics, 30(1), 182-191. doi:10.1080/10618600.2020.1806855
Helwig, N. E. (2024). Precise tensor product smoothing via spectral splines. Stats, 7(1), 34-53. doi:10.3390/stats7010003

\section*{See Also}

See rk for details on the reproducing kernel basis

\section*{Examples}
```


# load auto data

data(auto)

# additive effects

x <- rk.model.matrix(mpg ~ ., data = auto)
dim(x) \# check dimensions
attr(x, "assign") \# check group assignments
attr(x, "term.labels") \# check term labels

# two-way interactions

x <- rk.model.matrix(mpg ~ . * ., data = auto)
dim(x) \# check dimensions
attr(x, "assign") \# check group assignments
attr(x, "term.labels") \# check term labels

# specify df for horsepower, weight, and acceleration

# note: default df = 5 is used for displacement and model.year

df <- list(horsepower = 6, weight = 7, acceleration = 8)
x <- rk.model.matrix(mpg ~ ., data = auto, df = df)
sapply(attr(x, "knots"), length) \# check df

# specify knots for model.year

# note: default knots are selected for other variables

knots <- list(model.year = c(1970, 1974, 1978, 1982))
x <- rk.model.matrix(mpg ~ ., data = auto, knots = knots)
sapply(attr(x, "knots"), length) \# check df

```

\section*{Description}

Calculates the row-wise Kronecker product between two matrices with the same number of rows.

\section*{Usage}
row.kronecker (X, Y)

\section*{Arguments}
\(\mathrm{X} \quad\) matrix of dimension \(n \times p\)
\(\mathrm{Y} \quad\) matrix of dimension \(n \times q\)

\section*{Details}

Given \(X\) of dimension \(c(n, p)\) and \(Y\) of dimension \(c(n, q)\), this function returns
\(\operatorname{cbind}(x[, 1] * Y, x[, 2] * Y, \ldots, x[, p] * Y)\)
which is a matrix of dimension \(c(n, p * q)\)

\section*{Value}

Matrix of dimension \(n \times p q\) where each row contains the Kronecker product between the corresponding rows of X and Y .

\section*{Author(s)}

Nathaniel E. Helwig <helwig@umn.edu>

\section*{See Also}

Used by the rk. model.matrix to construct basis functions for interaction terms See kronecker for the regular kronecker product

\section*{Examples}
```

$\mathrm{X}<-\operatorname{matrix}(\mathrm{c}(1,1,2,2)$, nrow $=2$, ncol = 2)
$\mathrm{Y}<-\operatorname{matrix}(1: 6$, nrow $=2$, ncol $=3$ )
row.kronecker (X, Y)

```
StartupMessage Startup Message for grpnet

\section*{Description}

Prints the startup message when grpnet is loaded. Not intended to be called by the user.

\section*{Details}

The 'grpnet' ascii start-up message was created using the taag software.

\section*{References}
https://patorjk.com/software/taag/

\section*{Description}

Makes a plot or returns a data frame containing the group elastic net penalty (or its derivative) evaluated at a sequence of input values.

\section*{Usage}
```

visualize.penalty(x = seq(-5, 5, length.out = 1001),
penalty = c("LASSO", "MCP", "SCAD"),
alpha = 1,
lambda $=1$,
gamma = 4,
derivative = FALSE,
plot = TRUE,
subtitle = TRUE,
legend = TRUE,
location = ifelse(derivative, "bottom", "top"),
...)

```

\section*{Arguments}
x
penalty which penalty or penalties should be plotted?
alpha elastic net tuning parameter (between 0 and 1 ).
lambda overall tuning parameter (non-negative).
gamma additional hyperparameter for MCP (>1) or SCAD (>2).
derivative if FALSE (default), then the penalty is plotted; otherwise the derivative of the penalty is plotted.
plot if TRUE (default), then the result is plotted; otherwise the result is returned as a data frame.
subtitle if TRUE (default), then the hyperparameter values are displayed in the subtitle.
legend if TRUE (default), then a legend is included to distinguish the different penalty types.
location the legend's location; ignored if legend \(=\) FALSE.
\(\ldots \quad\) addition arguments passed to plot function, e.g., xlim, ylim, etc.
sequence of values at which to evaluate the penalty.

\section*{Details}

The group elastic net penalty is defined as
\[
P_{\alpha, \lambda}(\boldsymbol{\beta})=Q_{\lambda_{1}}(\|\boldsymbol{\beta}\|)+\frac{\lambda_{2}}{2}\|\boldsymbol{\beta}\|^{2}
\]
where \(Q_{\lambda}()\) denotes the L1 penalty (LASSO, MCP, or SCAD), \(\|\boldsymbol{\beta}\|=\left(\boldsymbol{\beta}^{\top} \boldsymbol{\beta}\right)^{1 / 2}\) denotes the Euclidean norm, \(\lambda_{1}=\lambda \alpha\) is the L1 tuning parameter, and \(\lambda_{2}=\lambda(1-\alpha)\) is the L 2 tuning parameter. Note that \(\lambda\) and \(\alpha\) denote the lambda and alpha arguments.

\section*{Value}

If plot \(=\) TRUE, then produces a plot.
If plot \(=\) FALSE, then returns a data frame .

\section*{Author(s)}

Nathaniel E. Helwig <helwig@umn.edu>

\section*{References}

Fan J, \& Li, R. (2001). Variable selection via nonconcave penalized likelihood and its oracle properties. Journal of the American Statistical Association, 96(456), 1348-1360. doi:10.1198/ 016214501753382273

Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. Journal of Computational and Graphical Statistics. doi:10.1080/ 10618600.2024.2362232

Tibshirani, R. (1996). Regression and shrinkage via the Lasso. Journal of the Royal Statistical Society, Series B, 58, 267-288. doi:10.1111/j.25176161.1996.tb02080.x

Zhang CH (2010). Nearly unbiased variable selection under minimax concave penalty. The Annals of Statistics, 38(2), 894-942. doi:10.1214/09AOS729

\section*{See Also}
visualize.shrink for plotting shrinkage operator

\section*{Examples}
```


# plot penalty functions

visualize.penalty()

# plot penalty derivatives

visualize.penalty(derivative = TRUE)

```

\section*{Description}

Makes a plot or returns a data frame containing the group elastic net shrinkage operator (or its estimator) evaluated at a sequence of input values.

\section*{Usage}
```

visualize.shrink(x = seq( $-5,5$, length.out $=1001$ ),
penalty = c("LASSO", "MCP", "SCAD"),
alpha = 1,
lambda $=1$,
gamma = 4,
fitted = FALSE,
plot $=$ TRUE,
subtitle = TRUE,
legend = TRUE,
location = "top",
...)

```

\section*{Arguments}
\(x\)
penalty
alpha elastic net tuning parameter (between 0 and 1 ).
lambda overall tuning parameter (non-negative).
gamma additional hyperparameter for MCP (>1) or SCAD (>2).
fitted if FALSE (default), then the shrinkage operator is plotted; otherwise the shrunken estimator is plotted.
plot if TRUE (default), then the result is plotted; otherwise the result is returned as a data frame.
subtitle if TRUE (default), then the hyperparameter values are displayed in the subtitle.
legend if TRUE (default), then a legend is included to distinguish the different penalty types.
location the legend's location; ignored if legend \(=\) FALSE.
\(\ldots \quad\) addition arguments passed to plot function, e.g., xlim, ylim, etc.
sequence of values at which to evaluate the penalty.
which penalty or penalties should be plotted?

\section*{Details}

The updates for the group elastic net estimator have the form
\[
\boldsymbol{\beta}_{\alpha, \lambda}^{(t+1)}=S_{\lambda_{1}, \lambda_{2}}\left(\left\|\mathbf{b}_{\alpha, \lambda}^{(t+1)}\right\|\right) \mathbf{b}_{\alpha, \lambda}^{(t+1)}
\]
where \(S_{\lambda_{1}, \lambda_{2}}(\cdot)\) is a shrinkage and selection operator, and
\[
\mathbf{b}_{\alpha, \lambda}^{(t+1)}=\boldsymbol{\beta}_{\alpha, \lambda}^{(t)}+\left(\delta_{(t)} \epsilon\right)^{-1} \mathbf{g}^{(t)}
\]
is the unpenalized update with \(\mathbf{g}^{(t)}\) denoting the current gradient.
Note that \(\lambda_{1}=\lambda \alpha\) is the L1 tuning parameter, \(\lambda_{2}=\lambda(1-\alpha)\) is the L 2 tuning parameter, \(\delta_{(t)}\) is an upper-bound on the weights appearing in the Fisher information matrix, and \(\epsilon\) is the largest eigenvalue of the Gramm matrix \(n^{-1} \mathbf{X}^{\top} \mathbf{X}\).

\section*{Value}

If plot \(=\) TRUE, then produces a plot.
If plot \(=\) FALSE, then returns a data frame.

\section*{Author(s)}

Nathaniel E. Helwig <helwig@umn.edu>

\section*{References}

Fan J, \& Li, R. (2001). Variable selection via nonconcave penalized likelihood and its oracle properties. Journal of the American Statistical Association, 96(456), 1348-1360. doi:10.1198/ 016214501753382273

Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. Journal of Computational and Graphical Statistics. doi:10.1080/ 10618600.2024.2362232

Tibshirani, R. (1996). Regression and shrinkage via the Lasso. Journal of the Royal Statistical Society, Series B, 58, 267-288. doi:10.1111/j.25176161.1996.tb02080.x
Zhang CH (2010). Nearly unbiased variable selection under minimax concave penalty. The Annals of Statistics, 38(2), 894-942. doi:10.1214/09AOS729

\section*{See Also}
visualize. penalty for plotting penalty function

\section*{Examples}
```


# plot shrinkage operator

visualize.shrink()

# plot shrunken estimator

visualize.shrink(fitted = TRUE)

```

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